

**ஜப்த்ச்தி; க்யி யஃஃஃ;**  
**THIRUVALLUVAR UNIVERSITY- VELLORE**

SYLLABUS- MATHEMATICS

COMMON ENTRANCE TEST FOR M.PHILL & PH.D

**UNIT – 1 ALGEBRA**

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Direct products, Finite abelian groups, Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Fields, finite fields, field extensions, Galois Theory. Vector spaces, algebra of linear transformations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Quadratic forms, reduction and classification of quadratic forms.

**UNIT – 2 ANALYSIS**

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano-Weierstrass theorem, Heine-Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation. Metric spaces, compactness, connectedness. Normed Linear Spaces. Spaces of Continuous functions as examples.

**UNIT-3 COMPLEX ANALYSIS**

The complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Möbius transformations.

## **UNIT-4 ORDINARY DIFFERENTIAL EQUATIONS (ODEs) & PARTIAL DIFFERENTIAL EQUATIONS (PDEs)**

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

## **UNIT -5 NUMBER THEORY AND DISCRETE MATHEMATICS**

Divisibility; Linear diophantine equations. Congruences. Quadratic residues; Sums of two squares, Arithmetic functions Mu, Tau, and Sigma

Partially ordered sets, Lattices, Complete Lattices, Distributive lattices, Complements, Boolean Algebra, Boolean Expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, planar Graphs, Directed Graphs, Trees, Permutations and Combinations, Pigeonhole principle, principle of Inclusion and Exclusion, Derangements.

## **UNIT-6 MECHANICS AND CALCULUS OF VARIATIONS**

The Mechanical system - Generalized coordinates - Holonomic and non-holonomic systems - constraints - Virtual work - D'Alembert's principle - Energy and Momentum. Derivation of Lagrange's equations - Examples - integrals of motion - cyclic or ignorable coordinates. Hamilton's principle - Hamilton's equations - other variational principle - Principle of Least action. Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema, Variation methods for boundary value problems in ordinary and partial differential equations. Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

## **UNIT-7 TOPOLOGY**

Topological Spaces - Examples - Basis for a topology - Sub-basis - closed sets - interior - closure - boundary - Limit points - Hausdorff spaces Subspace topology - Continuous functions - Examples - Homeomorphisms - topological property pasting lemma - the metric topology - Sequence Lemma - Uniform Limit theorem. Connected Spaces - components and Local connectedness - Totally disconnected spaces - Compact Spaces - The Lebesgue number lemma - Uniform continuity theorem - First countable and second countable spaces - separation axioms - regular and completely regular spaces - Normal and completely Normal spaces - Urysohn's lemma - Urysohn's metrization theorem - Tietze Extension theorem.

## **UNIT-8 FLUID DYNAMICS**

Real fluids and ideal fluids- velocity of a point- Stream lines and path lines – steady and unsteady flow – the velocity potential – the vorticity vector – local and particle rates of changes – the equation of continuity – worked examples- Pressure at a point in fluid at rest – Pressure at a point in a moving fluid – conditions at a boundary of two inviscid immiscible fluid – Euler's equation of motion – Bernoulli's equation – worked examples-sources – sinks and doublets – Axis symmetric flow – Stokes stream function. Meaning of two dimensional flows – use of cylindrical polar co-ordinates – the stream function – the potential for two dimensional – irrotational – incompressible flows – complex velocity potential for standard two dimensional flows – The Milne-Thomson Circle theorem. Stress components in real fluids – relation between Cartesian components of stress – translation motion of a fluid element some further properties of the rate of strain quadric stress analysis in fluid motion- the Navier – Stokes equation of motion of a viscous fluid.

## **UNIT-9 APPLIED NUMERICAL METHODS**

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

## **UNIT-10 APPLIED PROBABILITY AND STATISTICS**

Sample space, discrete probability, independent events, Bayes theorem, Random variables and distribution functions (univariate and multivariate), expectation and moments, Independent random variables, marginal and conditional distributions, Characteristic functions, Probability inequalities (Tchebyshef, Markov, Jensen), Central Limit theorems, Methods of estimation, properties of estimators, Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests, Analysis of discrete data and chi-square test of goodness of fit, rank correlation, best linear unbiased estimators, Analysis of variance and covariance, Simple and multiple linear regression, Ratio and regression methods, Multivariate normal distribution, partial and multiple correlation coefficients and related tests. Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD.